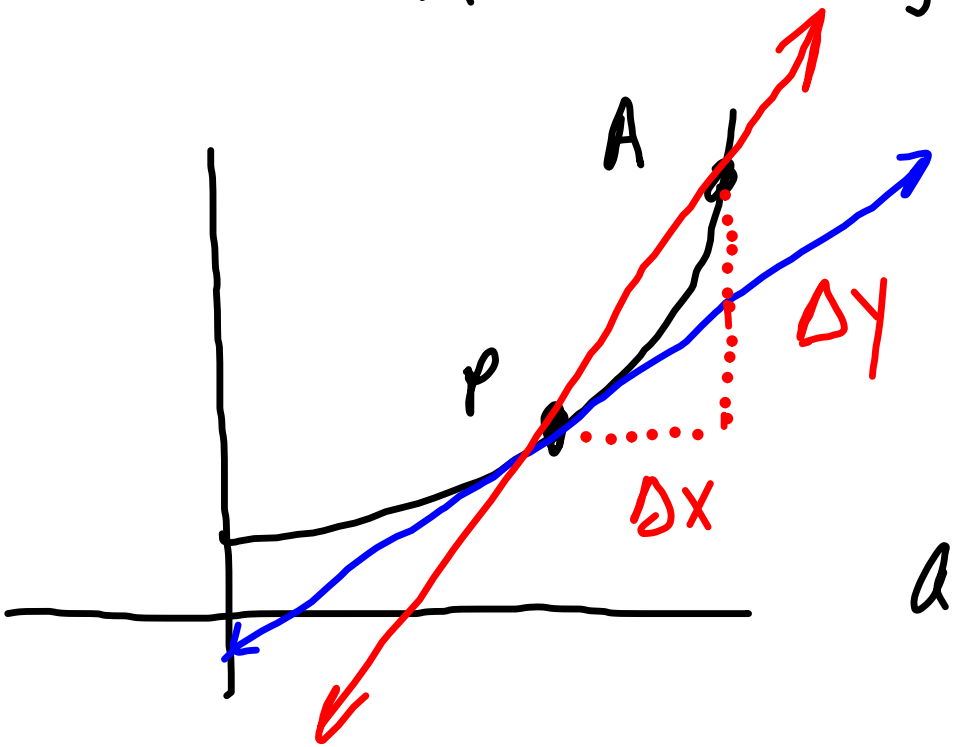
2.4 b tangent lines

Instantaneous rates of change



need: curve
point

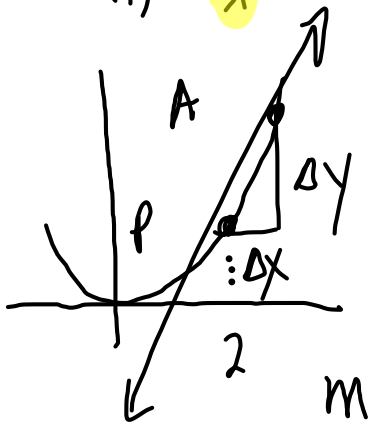
as $A \to P$,

$\frac{\Delta y}{\Delta x} \to$ slope of tan

$\Delta x, \Delta y \to 0$

$\frac{\Delta y}{\Delta x} \to m_{tan}$

$f(x) = x^2$ find slope of tan at $x=2$



$x_1 = 2 \quad y_1 = 4$

$x_2 = 2+\Delta x \quad y_2 = (2+\Delta x)^2$

$$m_{sec} = \frac{(2+\Delta x)^2 - 4}{2+\Delta x - 2}$$

simplify

$$= \frac{4+4\Delta x+\Delta x^2 - 4}{\Delta x}$$

factor

$$= \frac{\Delta x(4+\Delta x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} 4+\Delta x = 4$$

slope of the tan line

as $\Delta x \to 0$

$4+\Delta x \to 4$

find the eqn of the tan line

$m = 4$ point $(2, 4)$

eqn: $y = 4(x-2) + 4$

$y = m(x - x_1) + y_1$

normal line: $y = -\frac{1}{4}(x-2) + 4$

2.4 #9 $y = x^2$ $x_1 = -2$ $y_1 = 4$

$x_2 = -2 + \Delta x$

$y_2 = (-2+\Delta x)^2$

$$m_{sec} = \frac{(-2+\Delta x)^2 - 4}{-2+\Delta x - -2}$$

$$= \frac{4 - 4\Delta x + \Delta x^2 - 4}{\Delta x}$$

$$= \frac{\Delta x(-4+\Delta x)}{\Delta x}$$

$$m_{tan} = \lim_{\Delta x \to 0} -4 + \Delta x$$

$$= -4$$

tan line $y = -4(x+2)+4$

normal line $y = \frac{1}{4}(x+2)+4$

24. $y = 3t^2$ inst. vel at $t_1 = 10$

or slope of tan $y_1 = 300$

$t_2 = 10 + \Delta t$

$y_2 = 3(10 + \Delta t)^2$

$f(t) = \frac{1}{x-1}$ slope at $x_1 = 2$

$y_1 = 1$

$m_{sec} = \frac{\frac{1}{1+\Delta x} - 1\left(\frac{1+\Delta x}{1+\Delta x}\right)}{\Delta x}$ $x_2 = 2+\Delta x$

$y_2 = \frac{1}{2+\Delta x - 1} = \frac{1}{1+\Delta x}$

$= \frac{\frac{1}{1+\Delta x} - \frac{1+\Delta x}{1+\Delta x}}{\Delta x} = \frac{\frac{1-(1+\Delta x)}{1+\Delta x}}{\Delta x}$

$= \frac{\frac{-\Delta x}{1+\Delta x}}{\Delta x} = \frac{-\Delta x}{1+\Delta x} \cdot \frac{1}{\Delta x}$